Master on Physics and Engineering on Fusion Plasmas
Introductory Atomic and Molecular Physics. Problems (4).

1. Find the eigenfunctions of the harmonic oscillator Hamiltonian in the momentum representation.

2. Consider a system with two possible values of the angular momentum: 0 and $\sqrt{2}\hbar$. $\{|j\,m>\}$ is a basis of simultaneous eigenvectors of J^2 and J_z that fulfill:

$$J_\pm|j\,m>=\hbar\,[j(j+1)-m(m\pm 1]^{1/2}\,|j\,m\pm 1>$$

$$J_+|j\,j>=J_-|j\,-j>=0$$

   (a) Write the simultaneous eigenfunctions of J_x and J^2 in terms of the basis vectors $|j\,m>$.

   (b) The system is in a state $|\Psi>$ of the form:

$$|\Psi>=\alpha|1\,1>+\beta|1\,0>+\gamma|1\,-1>+\delta|0\,0>$$

      (i) Find the probablity of obtaining $2\hbar^2$ and $\hbar$ in a simultaneous measurement of J^2 and J_x.

      (ii) Evaluate $<J_z>$ and the probability of obtaining in a measurement each value of this observable.

3. Find the unitary matrix $\boldsymbol{S}$ that diagonalizes the matrix representation of J_x in the basis of simultaneous eigenvectors of J^2 and J_z with $j=1/2$. Compare $\boldsymbol{S}$ with the matrix representation of a rotation about the Y axis. Obtain the matrix representations of J_y, J_z and J^2 in the new basis.

4. (a) Show that a function $f(\boldsymbol{r})$ is transformed into

$$U_R f(\boldsymbol{r}) \simeq \left[1-\frac{\mathrm{i}}{\hbar}\boldsymbol{\phi}\cdot\boldsymbol{L}\right] f(\boldsymbol{r})$$

   by a rotation of the coordinate system, where $\boldsymbol{\phi}$ is a vector along the rotation axis whose modulus is the infinitesimal rotation angle.

   (b) Show that an infinitesimal rotation of the coordinate system transform a quantum mechanical operator F into

$$UFU^\dagger \simeq F-\frac{\mathrm{i}}{\hbar}\boldsymbol{\phi}\cdot[\boldsymbol{L},\,F]$$

   with $U=\left[1-\frac{\mathrm{i}}{\hbar}\boldsymbol{\phi}\cdot\boldsymbol{L}\right]$.

5. (a) Show that the Clebsch-Gordan coefficient $C(j_1, j_2, j, m_1, m_2, m)$ vanishes when $m_1 + m_2 \neq m$.

   (b) When the angular momentum vectors $\boldsymbol{j_1}, \boldsymbol{j_2}$ of two independent particles are added one obtains:

$$m_{\text{max}} = (m_1 + m_2)_{\text{max}}; \; j_{\text{max}} = j_1 + j_2$$

   Which is the value of j_{min}?

6. Build up a complete series of eigenfunctions of J^2 and J_z for a system of two independent particles with spin 1/2 as linear combinations of products of the corresponding functions for each particle.

7. Find the eigenfunctions of J^2 and J_z for a system of two independent particles with $j = 1$.